1.

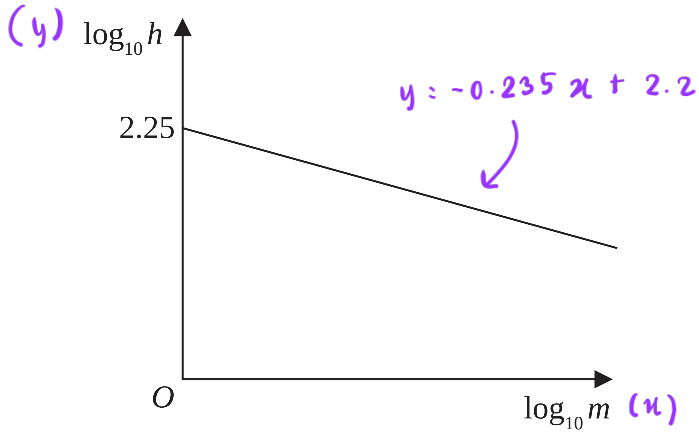


Figure 2

The resting heart rate, h, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of -0.235

(a) Find, to 3 significant figures, the value of p and the value of q.

(3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

a) As the relationship between $\log_{10} h$ and $\log_{10} m$ is linear,

we can write it as $y = mx + c$

where $x = \log_{10} m$, $y = \log_{10} h$, $m = -0.235$, $c = 2.25$

$\Rightarrow \log_{10} h = -0.235 \log_{10} m + 2.25$

$h = 10^{-0.235 \log_{10} m} \times 10^{2.25}$

$h = m^{-0.235} \times 10^{2.25}$ (1) $= pm^q$

$p = 10^{2.25}$ (1) $= 178$ and $q = -0.235$ (1)

b) If $m = 5$ kg, then the model predicts

$$h = 178 \times 5^{-0.235} \; (1) \; = 122 \; (1) \text{ beats per minute}$$

This is accurate to the measured heart rate within 2 significant figures. So, the model is suitable. (1)

c) p would be the resting heart rate in bpm of a mammal with a mass of 1 kg. (1)

2. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

(a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

(b) With reference to the model, interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find, according to the model,

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

(d) State one reason why this may not be a realistic model in the long term.

(1)

a) $\log_{10} A = 0.03t + 0.5$

$A = 10^{0.03t + 0.5}$ (1)

$A = 10^{0.03t} \times 10^{0.5}$ (1)

$A = (10^{0.5})(10^{0.03})^t$ (1)

p q

$\therefore A = 3.162 \times 1.072^t$ (4 s.f.) (1)

b)(i) p represents initial mass of algae (in kg), 0 weeks after the mass of algae was first recorded. 1

(ii) q represents the rate of growth of algae (in kg/week) 1

(c) when $t = 8$, find A

$A = (10^{0.5})(10^{0.03})^8$

$= 5.495$

$\therefore A = 5.5$ kg 1 (nearest 0.5 kg)

when $A = 4$, find t

$4 = (10^{0.5})(10^{0.03})^t$ 1

$10^{0.03t} = \frac{4}{10^{0.5}} = 1.26\ldots$

$0.03t = \log_{10} 1.26\ldots = 0.102\ldots$

$t = 3.401\ldots$

$= 3.4$ weeks 1

d) The small pond will soon be overcrowded, so it is unlikely for algae to multipy at the same rate. 1

3. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p,

(i) $\log_3\left(\frac{x}{9}\right)$

(ii) $\log_3\left(\sqrt{x}\right)$

(2)

(b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

a) (i) $\log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 9$

$= p - 2$ (1)

(ii) $\log_3 \sqrt{x} = \log_3 x^{\frac{1}{2}}$

$= \frac{1}{2}\log_3 x$

$= \frac{1}{2}p$ (1)

use value from (a)

b) $2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11$ (1)

$2(p-2) + 3\left(\frac{1}{2}p\right) = -11$

$2p - 4 + \frac{3}{2}p = -11$

$4p - 8 + 3p = -22$

$7p = -14$

$p = -2$ (1)

$\Rightarrow \quad p = \log_3 x$

$\log_3 x = -2$ (1)

$x = 3^{-2} = \frac{1}{9}$ (1)

4. Using the laws of logarithms, solve the equation

$$2\log_5(3x-2) - \log_5 x = 2$$

(5)

$2\log_5(3x-2) - \log_5 x = 2$

$\log_5(3x-2)^2 - \log_5 x = 2$ 1

$\log_5 \dfrac{(3x-2)^2}{x} = 2$ 1

$\dfrac{(3x-2)^2}{x} = 5^2$ 1

$9x^2 - 12x + 4 = 25x$ 1

$9x^2 - 37x + 4 = 0$

$(x-4)(9x-1) = 0$

$x = 4$ and $x = \frac{1}{9}$

substitute $x = \frac{1}{9}$ into the equation of log will give $\log_5$ of a negative number which is not correct. Hence, $x = \frac{1}{9}$ is not a solution.

$\therefore x = 4$ only 1

5. Using the laws of logarithms, solve the equation

$$\log_3(12y + 5) - \log_3(1 - 3y) = 2$$

(3)

$\log_3(12y+5) - \log_3(1-3y) = 2$ $\qquad \log_a b - \log_a c = \log_a \frac{b}{c}$

$\log_3\left(\frac{12y+5}{1-3y}\right) = 2$ 1

$\frac{12y+5}{1-3y} = 3^2$

$12y+5 = 9(1-3y)$

$12y+5 = 9-27y$ 1

$39y = 4$

$y = \frac{4}{39}$ 1

make sure to substitute $y = \frac{4}{39}$ into $12y+5$ and $1-3y$ to ensure they are positive and so the logarithms are valid.

6. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b\log_{10} l + \log_{10} a$$

(2)

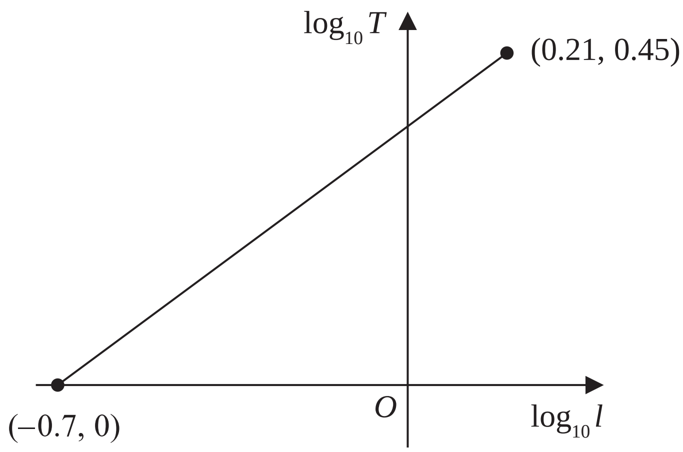


Figure 3

A student carried out an experiment to find the values of the constants a and b.

The student recorded the value of T for different values of l.

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data. The straight line passes through the points (–0.7, 0) and (0.21, 0.45)

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b, each to 3 significant figures.

(3)

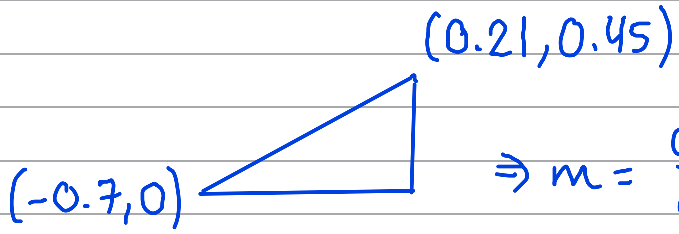
(c) With reference to the model, interpret the value of the constant a.

(1)

a) $T = al^b \Rightarrow \log_{10} T = \log_{10} al^b$

$\log_{10} T = \log_{10} a + \log_{10} l^b$ 1

$\log_{10} T = \log_{10} a + b\log_{10} l$ 1 b)

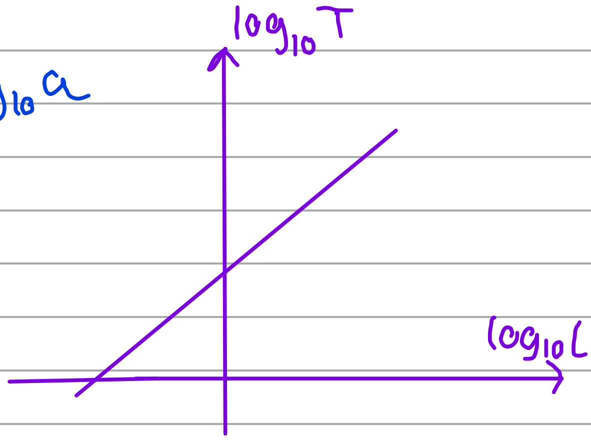


$\Rightarrow m = \dfrac{0.45-0}{0.21+0.7} = \dfrac{45}{91}$

we have $\log_{10}T = b\log_{10}l + \log_{10}a$

$\Rightarrow$ gradient $= b$

$\therefore b = \dfrac{45}{91}$ (1)



sub in $(-0.7, 0)$:

$0 = \dfrac{45}{91}(-0.7) + \log_{10}a$ (1)

$\log_{10}a = 0.346\ldots$

$a = 10^{0.346\ldots} = 2.218\ldots$

$T = 2.22l^{0.495}$ (1)

c) a is the time taken for a pendulum of length 1m to complete one full swing. (1)

7. (a) Sketch the curve with equation

$$y = 4^x$$

stating any points of intersection with the coordinate axes.

(2)

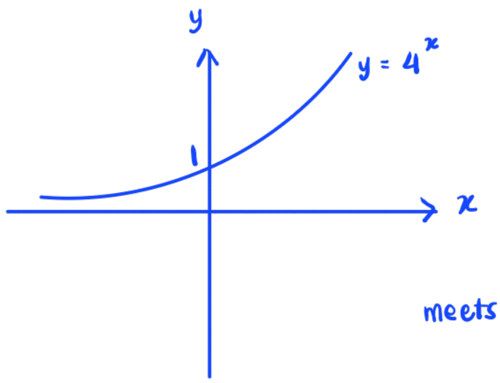
(b) Solve

$$4^x = 100$$

giving your answer to 2 decimal places.

(2)

a)



2

meets y-axis at (0,1)

b) $4^x = 100$

$\ln(4^x) = \ln(100)$

$x\ln(4) = \ln(100)$

$x = \dfrac{\ln(100)}{\ln(4)} = 3.321928$ 1

$x = 3.32$ (2 d.p.)
1

8. $a = \log_2 x \qquad b = \log_2(x + 8)$

Express in terms of a and/or b

(a) $\log_2 \sqrt{x}$ **(1)**

(b) $\log_2(x^2 + 8x)$ **(2)**

(c) $\log_2\left(8 + \frac{64}{x}\right)$

Give your answer in simplest form. **(3)**

$\log a^b = b \log a$

a) $\log_2 \sqrt{x} = \log_2 x^{1/2} = \frac{1}{2}\log_2 x = \frac{1}{2}a$ ①

b) $\log_2(x^2 + 8x) = \log_2(x(x+8)) = \log_2 x + \log_2(x+8)$ ①

$= a + b$ ①

c) $8 + \frac{64}{x} = \frac{8}{x}(x+8)$ ①

$\log(ab) = \log a + \log b$

$\log_2\left(8 + \frac{64}{x}\right) = \log_2\left(\frac{8}{x}(x+8)\right)$

$= \log_2 8 + \log_2(x+8) - \log_2 x$ ①

$= 3 + b - a$ ①

$\log \frac{a}{b} = \log a - \log b$

9.

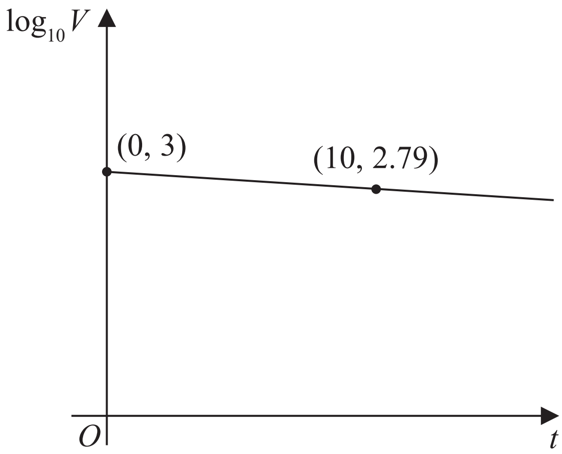


Figure 2

The value, V pounds, of a mobile phone, t months after it was bought, is modelled by

$$V = ab^t$$

where a and b are constants.

Figure 2 shows the linear relationship between $\log_{10} V$ and t.

The line passes through the points (0, 3) and (10, 2.79) as shown.

Using these points,

(a) find the initial value of the phone,

(2)

(b) find a complete equation for V in terms of t, giving the exact value of a and giving the value of b to 3 significant figures.

(3)

Exactly 2 years after it was bought, the value of the phone was £320

(c) Use this information to evaluate the reliability of the model.

(2)

a) when $t=0$, $\log_{10} V = 3$ ①

$\therefore V = 10^3 = £1000$ ①

b) $V = ab^t$

$\log_{10} V = \log_{10} ab^t$

$= \log_{10} a + \log_{10} b^t$

$= \log_{10} a + t \log_{10} b$

$\therefore$ gradient $= \log_{10} b$ (1)

$\log_{10} V$ intercept $= \log_{10} a$

$\log_{10} b = \dfrac{2.79 - 3}{10 - 0} = -0.021$ (1)

$\therefore \log_{10} a = 3$

$a = 1000$

$b = 10^{-0.021} = 0.953$ (3sf)

$V = 1000(0.953)^t$ (1)

c) 2 years = 24 months so sub in $t = 24$

$V = 1000(0.953)^{24} = £315$ (3sf) (1)

£315 is close to £320 so the model is suitable. (1)

10. Given that

$$\log_2(x + 3) + \log_2(x + 10) = 2 + 2\log_2 x$$

(a) show that

$$3x^2 - 13x - 30 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$3x^2 - 13x - 30 = 0$$

(ii) Hence state which of the roots in part (b)(i) is not a solution of

$$\log_2(x + 3) + \log_2(x + 10) = 2 + 2\log_2 x$$

giving a reason for your answer.

(2)

a) $\log_2(x+3) + \log_2(x+10) = 2 + 2\log_2 x$

$\log_2(x+3) + \log_2(x+10) - \log_2 x^2 = 2$

$\log_2\left(\frac{(x+3)(x+10)}{x^2}\right) = 2$ (1)

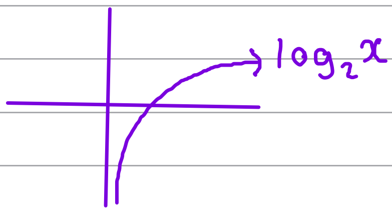
$\frac{x^2+13x+30}{x^2} = 2^2 = 4$

$x^2 + 13x + 30 = 4x^2$ (1)

$3x^2 - 13x - 30 = 0$ (1)

b) (i) $x = 6$, $x = -\frac{5}{3}$ (1)

(ii) $x \neq -\frac{5}{3}$ because $\log_2\left(-\frac{5}{3}\right)$ is not real. (1)



$\log x$ only takes positive inputs